IPMU11-0056

Hořava-Lifshitz gravity with $\lambda \to \infty$

A. Emir Gümrükçüoğlu and Shinji Mukohyama

Institute for the Physics and Mathematics of the Universe (IPMU)

The University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8582, Japan

(Dated: January 19, 2013)

Abstract

In the framework of the power-counting renormalizable theory of gravitation, recently proposed by Hořava, we study the limit $\lambda \to \infty$, which is arguably the most natural candidate for the ultraviolet fixed point of the renormalization group flow. In the projectable version with dynamical critical exponent $z = 3$, we analyze the Friedmann-Robertson-Walker background driven by the so-called "dark matter as integration constant" and perturbations around it. We show that amplitudes of quantum fluctuations for both scalar and tensor gravitons remain finite in the limit and that the theory is weakly coupled under a certain condition.

I. INTRODUCTION

A new theory of gravitation proposed recently by Hořava [1] has been attracting significant interest. (See [2, 3] for a review.) This theory, often called Hořava-Lifshitz (HL) gravity, is power-counting renormalizable thanks to the anisotropic scaling in the UV,

$$t \to b^z t, \quad \vec{x} \to b\vec{x}, \tag{1}$$

with the dynamical critical exponent $z \geq 3$.

The scaling (1) treats the time and the space in a different way. Hence, in order to realize this anisotropic scaling, the 4-dimensional diffeomorphism invariance cannot be a fundamental symmetry of the theory at high energy. Instead, the theory is invariant under the so-called foliation-preserving diffeomorphism:

$$t \to t'(t), \quad \vec{x} \to \vec{x}\,'(t, \vec{x}). \tag{2}$$

Because of this symmetry, the time kinetic Lagrangian for gravitons is a linear combination of K^2 and $K^{ij}K_{ij}$, where K_{ij} is the extrinsic curvature of constant-time hypersurfaces and $K = K^i_i$. Thus, the corresponding terms in the gravitational action are

$$I_g \ni \frac{M_{\rm Pl}^2}{2} \int N dt \sqrt{g} d^3\vec{x} \left(K^{ij}K_{ij} - \lambda K^2 \right), \tag{3}$$

where λ is a constant. In general relativity (GR) the 4-dimensional diffeomorphism invariance fixes the value of λ to 1. On the other hand, in HL gravity, any value of λ is consistent with the foliation-preserving diffeomorphism invariance.

HL gravity includes not only two degrees of freedom of usual tensor graviton but also one extra degree of freedom, dubbed the scalar graviton. The nature of this scalar degree depends on the value of the parameter λ. For $1/3 < \lambda < 1$, the scalar graviton has a wrong-sign time kinetic term (i.e. it is a ghost) and thus, this region is forbidden. For $\lambda < 1/3$ or $\lambda > 1$, the scalar graviton has a positive time kinetic term but has a negative sound speed squared, $c_s^2 = -(\lambda - 1)/(3\lambda - 1) < 0$ [4–6]. The condition under which the associated long-distance instability does not show up is [2]

$$0 < \frac{\lambda - 1}{3\lambda - 1} < \max\left[\frac{a^2 H^2}{k^2}, |\Phi|\right] \quad \text{for} \quad H < \frac{k}{a} < \min\left[M_s, \frac{1}{0.01\,\text{mm}}\right], \tag{4}$$

where k is the comoving momentum scale of interest, a is the scale factor, H is the Hubble expansion rate of the background cosmology, M_s is the energy scale at which the anisotropic

scaling becomes important for the scalar graviton, and we have introduced the Newtonian potential Φ by $M_{\rm Pl}^2\,(k/a)^2\,\Phi \sim -\rho$. Here, ρ is the energy density of the background. This condition essentially says that λ must be sufficiently close to 1 in the infrared (IR).

The condition (4) should be considered as a phenomenological constraint on properties of the renormalization group (RG) flow since λ is subject to running under the RG flow and in general, should depend on k, H and Φ. This suggests that, in order for the theory to be phenomenologically viable, $\lambda = 1$ should be an IR fixed point of the RG flow and that λ should approach 1 from above sufficiently quickly as the energy scale of the system is lowered. In this sense, $\lambda = 1 + 0$ is a candidate for the IR fixed point of the RG flow. Since the interval $1/3 < \lambda < 1$ is forbidden, a natural candidate for the UV fixed point that is consistent with the arguments for the IR fixed point above, is $\lambda = +\infty$.

The goal of this paper is to investigate some properties of the projectable version of the theory without detailed balance, in the vicinity of the expected UV fixed point, $\lambda = +\infty$. One might expect a loss of theoretical control in this limit since the coupling constant diverges. On the contrary, we show below that the theory is totally well-behaved and actually simpler in this limit.

The rest of this paper is organized as follows. In Sec. II, we review the basic equations in HL gravity with projectability condition. In Sec. III we discuss the background evolution of a FRW geometry describing our local patch of the universe populated by a perfect fluid. In Sec. IV we discuss the dynamics of tensor and scalar perturbations around the FRW universe. We conclude with Sec. V where we summarize our results and discuss some of the standing issues. A simple system of a Lifshitz scalar in HL gravity is investigated in the Appendix A.

II. HOŘAVA-LIFSHITZ GRAVITY: REVIEW AND BASIC EQUATIONS

HL gravity, being a less restricted theory than GR, requires the temporal and spatial coordinates to be treated on different grounds. The theory itself is invariant under the so-called foliation-preserving diffeomorphism, which is a combination of global time reparametrizations and spatial diffeomorphisms, characterized by the following infinitesimal transformations

$$\delta t = f(t)\,, \quad \delta x^i = \xi^i(t,\vec{x})\,. \tag{5}$$

Due to the different scaling dimensions of time and space coordinates, the 4-dimensional spacetime metric is not a fundamental quantity. Instead, fundamental quantities in the HL gravity are the lapse function $N(t)$, the shift vector $N^i(t,\vec{x})$ and the 3-dimensional spatial metric $g_{ij}(t,\vec{x})$. It is still useful, at least at low energies, to combine them into a 4-dimensional metric in the fashion of ADM [7],

$$ds^2 = -N^2\, dt^2 + g_{ij}\, \left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right) . \tag{6}$$

Note that the shift vector N^i and spatial metric g_{ij} depend on all four coordinates but that the lapse function N is assumed to be a function of time only. The latter assumption, dubbed *the projectability condition* is consistent with the foliation preserving diffeomorphism in the sense that a projectable N is mapped to another projectable N.

Starting with the time kinetic action (3), the most general gravitational action that respects the symmetries of the theory can be constructed as

$$I_g = \frac{M_{\rm Pl}^2}{2}\int N\, dt\, \sqrt{g}\, d^3\vec{x}\, \left(K^{ij}K_{ij} - \lambda K^2 - 2\Lambda + R + L_{z>1}\right), \tag{7}$$

where

$$K_{ij} \equiv \frac{1}{2\,N}\left(\dot{g}_{ij} - D_i N_j - D_j N_i\right) \tag{8}$$

is the extrinsic curvature and its trace K is obtained with its contraction with the 3d induced metric. The broken Lorentz symmetry manifests itself as an arbitrary parameter λ, which acquires the value 1 in GR. In the above action, we fixed the coefficient of the scalar curvature to unity by a choice of unit so that the Einstein-Hilbert action is reproduced in the IR limit with $\lambda \to 1$. Finally, the part of the action denoted by $L_{z>1}$ contains the higher spatial derivative terms and controls the UV behavior of the system. For definiteness, we will focus on the case with $z = 3$ scaling in the UV in the remainder of this paper. If the detailed balance is not enforced but if the spatial parity and time reflection symmetries are imposed, this choice allows spatial derivative terms up to sixth order as

$$\begin{aligned}\frac{M_{Pl}^2}{2}L_{z>1} = &\left(c_1\, D_i R_{jk} D^i R^{jk} + c_2\, D_i R\, D^i R + c_3 R_i^j R_j^k R_k^i + c_4 R\, R_i^j R_j^i + c_5 R^3\right)\\ &+ \left(c_6 R_i^j R_j^i + c_7 R^2\right) ,\end{aligned} \tag{9}$$

where D_i is the covariant derivative with respect to the 3d metric and c_i are constants.

The effect of matter on the dynamics is provided by the additional action term I_m, which is also required to be invariant under the foliation-preserving diffeomorphism.

By variation with respect to $g_{ij}(t,x)$, we obtain the dynamical equation

$$\mathcal{E}_{gij} + \mathcal{E}_{mij} = 0, \tag{10}$$

where

$$\mathcal{E}_{gij} \equiv g_{ik} g_{jl} \frac{2}{N\sqrt{g}} \frac{\delta I_g}{\delta g_{kl}}, \quad \mathcal{E}_{mij} \equiv g_{ik} g_{jl} \frac{2}{N\sqrt{g}} \frac{\delta I_m}{\delta g_{kl}} = T_{ij}. \tag{11}$$

Note that the matter sector (as well as the gravity sector) should be invariant under spatial diffeomorphism (as a part of the foliation preserving diffeomorphism) and thus it makes sense to define T_{ij} in general. The explicit expression for $\mathcal{E}_{gij}$ is given by

$$\begin{aligned}
\mathcal{E}_{gij} &= M_{Pl}^2 \Big[-\frac{1}{N}(\partial_t - N^k D_k) p_{ij} + \frac{1}{N}(p_{ik} D_j N^k + p_{jk} D_i N^k) \\
&\quad - K p_{ij} + 2K_i^k p_{kj} + \frac{1}{2} g_{ij} K^{kl} p_{kl} + \frac{1}{2}\Lambda g_{ij} - G_{ij} \Big] + \mathcal{E}_{z>1ij}, \\
p_{ij} &\equiv K_{ij} - \lambda K g_{ij},
\end{aligned} \tag{12}$$

where $\mathcal{E}_{z>1ij}$ is the contribution from $L_{z>1}$ and G_{ij} is Einstein tensor of g_{ij}.

Variation with respect to the shift $N^i(t,x)$ leads to the momentum constraint

$$\mathcal{H}_{gi} + \mathcal{H}_{mi} = 0, \tag{13}$$

where

$$\mathcal{H}_{gi} \equiv -\frac{\delta I_g}{\delta N^i} = -M_{Pl}^2 \sqrt{g} D^j p_{ij}, \quad \mathcal{H}_{mi} \equiv -\frac{\delta I_m}{\delta N^i}. \tag{14}$$

The only remaining equation is the Hamiltonian constraint, obtained by variation with respect to the lapse function $N(t)$,

$$H_{g\perp} + H_{m\perp} = 0, \tag{15}$$

where

$$H_{g\perp} \equiv -\frac{\delta I_g}{\delta N} = \int d^3\vec{x} \mathcal{H}_{g\perp}, \; H_{m\perp} \equiv -\frac{\delta I_m}{\delta N}, \tag{16}$$

and

$$\mathcal{H}_{g\perp} = \frac{M_{Pl}^2}{2} \sqrt{g} (K^{ij} p_{ij} - \Lambda - R - L_{z>1}). \tag{17}$$

Here, we stress that due to the projectability condition, which restricts N to be only time dependent, the Hamiltonian constraint in HL gravity is a global one, in contrast to the local one in GR.

Just for comparison, in a Lorentz invariant theory, the energy-momentum tensor is defined as

$$T_{\mu\nu}^{(\mathrm{LI})} = -\frac{2}{\sqrt{-^{(4)}g}}\frac{\delta I_m^{(\mathrm{LI})}}{\delta g^{\mu\nu}}\,, \tag{18}$$

and thus the matter terms in the constraints are expressed as

$$H_{m\perp} = \int d^3\vec{x}\sqrt{g}\; T_{\mu\nu}^{(LI)} n^\mu n^\nu, \quad \mathcal{H}_{mi} = \frac{1}{\sqrt{g}} T_{i\mu}^{(LI)} n^\mu, \tag{19}$$

where we defined the 4-vector n^μ to be the unit vector normal to the constant-time hypersurfaces, with

$$n^\mu \partial_\mu \equiv \frac{1}{N}\left(\partial_t - N^i\,\partial_i\right)\,. \tag{20}$$

III. FRW BACKGROUND

A Friedmann-Robertson-Walker (FRW) metric,

$$ds^2 = -dt^2 + a(t)^2\, d\vec{x}^2\,, \tag{21}$$

is supposed to describe the large-scale, overall behavior of the geometry in our local patch of the universe. Since the universe far outside the present horizon may be very different from the local universe inside the horizon, we should not expect the same FRW geometry to describe the whole spacetime including the region far outside our local patch [1]. Nonetheless, in general relativity, since the Hamiltonian constraint is a local equation satisfied at each spatial point, it leads to a Friedmann equation applicable to our local patch of the universe. On the other hand, in HL gravity with the projectability condition, the Hamiltonian constraint is a global equation integrated over the whole space. For this reason, the Hamiltonian constraint in HL gravity does not tell anything useful about the "local" FRW geometry [8].

Therefore, in HL gravity with the projectability condition, we do not have a Friedmann equation applicable to our local FRW universe. Instead, we have the dynamical equation (10) in the form

$$-\frac{3\,\lambda-1}{2}\left(2\,\dot{H} + 3\,H^2\right) = \frac{P}{M_{\mathrm{Pl}}^2}\,, \tag{22}$$

[1] This is in accord with the so called gradient expansion approach to super-horizon nonlinear cosmological perturbations [12].

where, in accord with the “local” homogeneity and isotropy of the “local” FRW geometry, we have assumed that the stress tensor of matter is “locally” homogeneous and isotropic as

$$T_{ij} = P(t) g_{ij} \quad \mathcal{H}_{mi} = 0 \,. \tag{23}$$

For the same reason as why we do not have a Friedmann equation applicable to a “local” FRW universe, we do not have a conservation equation for the “locally” homogeneous and isotropic matter. Therefore, we define a quantity $Q(t)$ by [2]

$$\dot{\rho} + 3\,H\,(\rho + P) = -Q \,, \tag{24}$$

where $H \equiv \dot{a}/a$. Note that Q is generically nonzero at high energies. From Eqs.(22) and (24), one can find a generalized Friedmann equation [2]

$$3\,M_{\rm Pl}^2\,H^2 = \rho_{''{\rm dm}''} + \frac{2}{3\,\lambda - 1}\,\rho \,, \tag{25}$$

where

$$\rho_{''{\rm dm}''} \equiv \frac{1}{a^3}\left[C_0 + \frac{2}{3\,\lambda - 1}\int_{t_0}^{t} Q(t')\,a^3(t')dt'\right] \,, \tag{26}$$

with C_0 an integration constant. The quantity $\rho_{''{\rm dm}''}$ is the “dark matter as integration constant”, associated with the lack of a local Hamiltonian constraint in HL gravity. See ref. [8] for more general cases.

Equation of motion of matter leads to conservation equation at least at low energy, provided that the local Lorentz invariance is restored in the matter sector as required by many experimental and observational data. In this case, we have $Q \to 0$ as $a \to \infty$, and the integral part of Eq.(26) converges to a constant. Thus at low energy, the $\rho_{''{\rm dm}''}$ component redshifts like nonrelativistic matter, or pressureless dust.

Let us now consider the limit $\lambda \to +\infty$. The dynamical equation (22) is reduced to

$$2\,\dot{H} + 3\,H^2 = 0 \,, \tag{27}$$

and the generalized Friedmann equation (25) is greatly simplified as

$$3\,M_{\rm Pl}^2\,H^2 = \rho_{''{\rm dm}''} \,, \quad \rho_{''{\rm dm}''} \equiv \frac{C_0}{a^3} \,. \tag{28}$$

[2] In HL gravity the projectability condition implies that we do not have to define a “local” energy density ρ. Nonetheless, just for our convenience we can still define ρ by pretending as if N were a function of time and spatial coordinates. With ρ defined in this way, the quantity Q measures the amount of deviation from what we would expect in theories with 4-dimensional spacetime diffeomorphism.

This shows that the matter sector decouples from the gravity sector and that the evolution of the local FRW universe is dominated by the "dark matter as integration constant" in the limit $\lambda \to \infty$.

However, from the cosmological viewpoint, we need to specify what we exactly mean by the limit $\lambda \to \infty$. Supposing that $\lambda = +\infty$ is a UV fixed point of the RG flow, the second term in the r.h.s. of (25) is indeed suppressed by $1/\lambda$ in the early universe. A similar suppression of coupling to the matter sector can be observed for the integral term in (26). However, the increase in λ going earlier in time does not necessarily imply that the matter sector is decoupled from gravity since ρ (and Q) also becomes large in the early universe. In order to obtain the decoupled equation (28), what we really have to ensure is that

$$\frac{\lambda\, \rho_{''\mathrm{dm}''}}{\rho} \simeq \frac{\lambda\, C_0}{\rho\, a^3} \gg 1. \tag{29}$$

Assuming logarithmic running of the coupling $\lambda \sim \log(H/M)$ for $H \gg M$, if the fluid energy redshifts faster than a^{-3}, the fluid generically dominates the expansion in the asymptotic past. On the other hand, even if the fluid dominates the expansion early on, the "dark matter" energy can in principle catch up later and become the dominant source while the theory is still in the UV regime. In this case, even though the earlier evolution exhibits a coupled behavior, the modes that are deep inside the horizon at the time of transition will not carry any memory of this early behavior. We will focus on scales for which at the time of (sound) horizon crossing the UV behavior $\lambda \gg 1$ is still valid and the fluid contribution to the expansion is suppressed relative to the "dark matter" as in (29). Any lengths beyond this scale are assumed to be well beyond the current observable universe.

As an alternative case, we can also consider a situation in which the fluid is pressure-less. In this case, the ratio $\lambda\, \rho_{''\mathrm{dm}''}/\rho$ grows logarithmically in the UV direction, and the matter indeed decouples from geometry in the asymptotic past [3].

[3] For time scales in which the logarithmic running of λ is not appreciable, there is no distinction between this pressure-less fluid and the "dark matter as integration constant" at the background level, since both have the same equation of state. On the other hand, at the level of perturbations, they are distinct even without taking into account the running of λ since the rest frame of the "dark matter" (but not that of the fluid) is synchronized with the spacetime foliation and dispersion relations for all physical degrees of freedom are associated with this foliation [9].

IV. PERTURBATION

In this section, motivated by the decoupling between gravity and matter in the limit $\lambda \to \infty$ observed in the previous section, we study a pure gravity system and analyze the evolution of perturbations around the FRW background driven by the "dark matter as integration constant". (In Appendix A, in order to justify this treatment we consider a scalar field in HL gravity and show that gravity and matter are decoupled in the limit $\lambda \to \infty$ for linear perturbations.) We investigate the UV regime with the dynamical critical exponent $z = 3$ and show that the amplitude of quantum fluctuations remains finite and that the system is well behaved in the $\lambda \to \infty$ limit.

One of the most important properties of HL gravity is the anisotropic scaling (1) with $z \geq 3$ since the power-counting renormalizability stems from it. Intriguingly, with the minimal value $z = 3$, this scaling can lead to a mechanism to generate scale-invariant cosmological perturbations even without inflation [10]. Let us briefly review this mechanism before going into the detailed analysis of perturbations.

With $z = 3$, we would like to know the condition for generation of super-horizon cosmological perturbations. Generation of super-horizon cosmological perturbation is nothing but oscillation followed by freeze-out. Each mode oscillates for $\omega^2 \gg H^2$ and freezes out for $\omega^2 \ll H^2$, where ω is the frequency of a mode of interest and $H = \dot{a}/a$ is the Hubble expansion rate. Thus, the condition for generation of cosmological perturbations is $\partial_t(H^2/\omega^2) > 0$. With the dispersion relation $\omega^2 \simeq (\vec{k}^2/a^2)^3/M^4$ expected from the $z = 3$ scaling, where $\vec{k}$ is the comoving momentum and M is a characteristic mass scale, this condition is reduced to $\partial_t^2(a^3) > 0$ for an expanding universe. This condition can be satisfied by, for example, a power-law expansion $a \propto t^p$ with $p > 1/3$, and does not require accelerated expansion ($p > 1$), i.e. inflation.

For concreteness, let us consider a scalar field ϕ with a canonical time kinetic term. The anisotropic scaling (1) implies that ϕ should scale as

$$\phi \to b^{-s}\phi, \quad s = \frac{3-z}{2}. \tag{30}$$

From this, it is expected that the amplitude of quantum fluctuations in a FRW background should be

$$\delta\phi \sim M \times \left(\frac{H}{M}\right)^{\frac{3-z}{2z}}, \tag{31}$$

where M is a characteristic mass scale in the action for ϕ, e.g. the scale suppressing higher spatial derivative terms. This reproduces the well-known result $\delta\phi \sim H$ for Lorentz invariant theories ($z = 1$) and $\delta\phi \sim (M^3 H)^{1/4}$ for ghost inflation [11] ($z = 2$). For HL gravity with $z = 3$, we have a Hubble-independent result, $\delta\phi \sim M$. Thus the amplitude of quantum fluctuations is expected to be scale-invariant in HL gravity with $z = 3$. This also applies to both tensor graviton and scalar graviton.

While $\delta\phi \sim M$ is generically expected for the HL gravity with $z = 3$, a numerical coefficient in front of M in the right hand side may depend on λ. It is not a priori clear whether this numerical coefficient remains finite or diverges when the $\lambda \to \infty$ limit is taken. In the following, we shall explicitly show that the amplitudes for tensor and scalar gravitons indeed remain finite in this limit.

We shall also investigate nonlinear interactions among tensor and scalar gravitons and show that the system remains weakly coupled in the UV with $\lambda \to \infty$, provided that

$$-c_1 \gg M_{\rm Pl}^{-2}, \quad -(3c_1 + 8c_2) \gg M_{\rm Pl}^{-2}. \tag{32}$$

Since c_1, c_2 and $M_{\rm Pl}^2$ are subject to running under the RG flow, this should be considered as a nontrivial condition on properties of the RG flow in the UV.

A. Tensor Modes

We now consider a pure gravity system and analyze tensor perturbations around the FRW background driven by the "dark matter as integration constant". Let us consider metric perturbations of the form

$$\delta N = 0, \quad \delta N_i = 0, \quad \delta g_{ij} = a^2 h_{ij}\,, \tag{33}$$

where h_{ij} is a transverse and traceless 3d tensor. The part of the gravitational action (10) containing the terms quadratic in tensor degrees can be obtained as

$$I_g \ni \frac{M_{\rm Pl}^2}{8}\int dt\, d^3\vec{x}\, a^3 \delta^{ik}\delta^{jl}\left[\dot{h}_{ij}\dot{h}_{kl} + h_{ij}\,\mathcal{O}_t\, h_{kl}\right]\,. \tag{34}$$

In the above action, the spatial derivatives are contained in the operator

$$\mathcal{O}_t \equiv \frac{1}{a^2}\triangle - \frac{\kappa_t}{a^4 M_t^2}\triangle^2 + \frac{1}{a^6 M_t^4}\triangle^3\,, \tag{35}$$

where $\triangle \equiv \delta^{ij}\partial_i\partial_j$ and M_t is some characteristic energy scale defined through

$$\frac{1}{M_t^4} \equiv -2\frac{c_1}{M_{\rm Pl}^2}\,, \qquad \frac{\kappa_t}{M_t^2} \equiv -2\frac{c_6}{M_{\rm Pl}^2}\,. \tag{36}$$

We remind the reader that in Eq.(35), the coefficient for the linear $\triangle/a^2$ term has already been fixed by a choice of unit (see discussion after (7)). Furthermore, we constrain the sign of the $\triangle^3$ term so that the evolution of the mode is stable in the UV at the asymptotic past and a vacuum state can be unambiguously defined. Finally, we do not restrict the sign of κ_t, but assume it is of order 1 in the following just for simplicity.

We now proceed with the quantization of the tensor mode by first expanding the tensor degrees in Fourier space as

$$h_{ij}(t,\vec{x}) = \frac{1}{(2\,\pi)^{3/2}} \sum_{\sigma=1,2} \int d^3k\, {\rm e}^{i\,\vec{k}\cdot\vec{x}}\, \epsilon_{ij}^{\sigma}(\vec{k})\, \hat{h}_\sigma(t,\vec{k})\,, \tag{37}$$

where ϵ_{ij}^{σ} are the transverse-traceless polarization tensors and σ can take values 1 or 2. It is convenient to introduce a new time parameterization as

$$dy \equiv \omega_t\, dt\,, \tag{38}$$

where ω_t is the frequency of the form

$$\omega_t^2 \equiv \frac{k^2}{a^2} + \frac{\kappa_t\, k^4}{a^4\, M_t^2} + \frac{k^6}{a^6\, M_t^4}\,. \tag{39}$$

This brings the kinetic part of the action (34) to

$$I_g \ni \frac{M_{\rm Pl}^2}{8} \sum_{\sigma} \int dy\, d^3k\, a^3\, \omega_t\, \hat{h}_\sigma^{\dagger\prime}\, \hat{h}_\sigma'\,, \tag{40}$$

where prime denotes differentiation with respect to the new time y. Expanding the operator $\hat{h}$ in a creation/annihilation operator basis as

$$\hat{h}_\sigma(\vec{k}) \equiv h_\sigma(k)\, \hat{a}_\sigma(\vec{k}) + h_\sigma^*(k)\, \hat{a}_\sigma^\dagger(-\vec{k})\,, \tag{41}$$

we introduce the mode functions that give rise to a canonically normalized kinetic action

$$\bar{h}_\sigma \equiv \frac{M_{\rm Pl}\, a^{3/2}\, \sqrt{\omega_t}}{2}\, h_\sigma\,, \tag{42}$$

obeying the equation of motion

$$\bar{h}_\sigma'' + \left[1 + \frac{1}{\omega_t^2}\, \left(\frac{3\,\dot{\omega}_t}{4\,\omega_t^2} - \frac{\ddot{\omega}_t}{2\,\omega_t}\right)\right] \bar{h}_\sigma = 0\,. \tag{43}$$

Noting that the frequency decreases as a^{-3} in the UV and the pure gravity background satisfies (27), the above equation becomes simply

$$\bar{h}''_\sigma + \bar{h}_\sigma = 0\,. \tag{44}$$

Fixing their amplitude from the kinetic part of the action and requiring that the corresponding state should minimize the quadratic Hamiltonian of the system [10], the mode functions of the tensor field can be written as

$$h_\sigma = \frac{\sqrt{2}}{\sqrt{\omega_t}\,M_{\rm Pl}\,a^{3/2}}\,{\rm e}^{-i\,y} \simeq \frac{\sqrt{2}M_t}{k^{3/2}\,M_{\rm Pl}}\,{\rm e}^{-i\,y}\,. \tag{45}$$

Through the two-point function, we define the tensor power spectrum P_t as

$$\sum_\sigma \left\langle \hat{h}_\sigma(t,\vec{k})\hat{h}^\dagger_\sigma(t,\vec{k}')\right\rangle \equiv \frac{2\,\pi^2}{k^3}\,\delta^{(3)}(\vec{k}-\vec{k}')P_t\,. \tag{46}$$

The power spectrum of the modes both sub and super horizon in the UV epoch turns out to be both scale invariant and time independent

$$P_t = \frac{k^3}{2\,\pi^2}\sum_\sigma |h_\sigma|^2 = \frac{2}{\pi^2}\left(\frac{M_t}{M_{\rm Pl}}\right)^2\,. \tag{47}$$

B. Scalar Modes

In this subsection, we discuss the evolution of scalar perturbations. The metric tensor has three local (i.e. space-dependent) and one global (i.e. space-independent) scalar degrees of freedom

$$\delta N = A\,,\quad \delta N_i = \partial_i B\,,\quad \delta g_{ij} = a^2\left[2\,\delta_{ij}\,\zeta + \partial_i\partial_j h_L\right]\,, \tag{48}$$

where $A = A(t)$ depends only on time in accordance with the projectability condition discussed in Sec.II. We fix the two scalar gauge degrees of freedom by setting $A = h_L = 0$. In this convenient gauge, the momentum constraint (13) reads [4]

$$B = \frac{3\,\lambda-1}{\lambda-1}\,\frac{\dot{\zeta}}{a^{-2}\triangle}\,, \tag{49}$$

[4] Although up to this point, we did not make any assumption on the details of the background evolution or the value of the constant λ, the linear equations in this section do not cover the case $\lambda = 1$ due to infinities arising in some of the relations, e.g. Eq.(49). However, the existence of such issues does not necessarily imply that the theory is not continuously connected to $\lambda = 1$ limit, but it is merely a manifestation that the perturbative expansion breaks down. The concrete study of the continuity requires a nonlinear analysis and is beyond the scope of the present paper. See the discussion in Sec.V for further comments on this issue.

while the equations of motion (10) leads to

$$\ddot{\zeta} + 3\,H\,\dot{\zeta} - \mathcal{O}_s\zeta = 0\,, \tag{50}$$

where we defined the operator

$$\mathcal{O}_s \equiv \frac{\lambda - 1}{3\,\lambda - 1}\left(-\frac{1}{a^2}\,\triangle - \frac{\kappa_s}{a^4\,M_s^2}\,\triangle^2 + \frac{1}{a^6\,M_s^4}\,\triangle^3\right)\,, \tag{51}$$

with

$$\frac{1}{M_s^4} \equiv -2\frac{3\,c_1 + 8\,c_2}{M_{\rm Pl}^2}\,, \qquad \frac{\kappa_s}{M_s^2} \equiv -2\frac{3\,c_6 + 8\,c_7}{M_{\rm Pl}^2}\,. \tag{52}$$

Proceeding as in previous subsection, we expand the scalar degrees in Fourier space, for which Eq.(50) becomes

$$\ddot{\hat{\zeta}} + 3\,H\,\dot{\hat{\zeta}} + \omega_s^2\hat{\zeta} = 0\,, \tag{53}$$

and the frequency of the scalar graviton perturbation is defined as

$$\omega_s^2 \equiv \frac{\lambda - 1}{3\,\lambda - 1}\left(-\frac{k^2}{a^2} + \frac{\kappa_s\,k^4}{a^4\,M_s^2} + \frac{k^6}{a^6\,M_s^4}\right)\,. \tag{54}$$

The form of the scalar mode frequency implies that at early times, ω_s^2 is dominated by the (positive) term proportional to k^6 and modes are in an oscillatory regime, much like the tensor modes discussed in the previous subsection. On the other hand, the frequency at late times may become dominated by the (negative) k^2 term, creating a ground for a linear instability. However, this happens after the Hubble friction takes over, so the time scale of this instability is not short enough to have an effect on the evolution. See Eq.(4) for the more general condition under which the long-distance instability does not show up.

We now proceed with the quantization of the scalar graviton degree. Under time parameterization $dy \equiv \omega_s dt$, the kinetic part of the scalar action reduces to

$$I_g \ni M_{\rm Pl}^2\left(\frac{3\,\lambda - 1}{\lambda - 1}\right)\int dy\,d^3k\,a^3\,\omega_s\,\hat{\zeta}^{\dagger\prime}\,\hat{\zeta}'\,. \tag{55}$$

The mode function for the canonical field can then be defined through

$$\bar{\zeta} \equiv \sqrt{2\,\omega_s}\sqrt{\frac{3\,\lambda - 1}{\lambda - 1}}\,a^{3/2}\,M_{\rm Pl}\,\zeta\,, \tag{56}$$

with equation of motion

$$\bar{\zeta}'' + \left[1 + \frac{1}{\omega_s^2}\left(\frac{3\,\dot{\omega}_s}{4\,\omega_s^2} - \frac{\ddot{\omega}_s}{2\,\omega_s}\right)\right]\bar{\zeta} = 0\,. \tag{57}$$

The time dependence of the frequency ω_s^2 is qualitatively the same as that of the tensor modes. In the UV regime, where one can approximate $\dot{\omega}_s \simeq -3\,H\,\omega_s$, one obtains a simple equation for the mode functions

$$\bar{\zeta}'' + \bar{\zeta} = 0\,. \tag{58}$$

The canonical scalar modes evolve qualitatively the same as the tensor modes in (44). By going from the canonical mode to the physical one, the solution for the scalar mode function can be written as

$$\zeta = \frac{1}{2\,M_{\rm Pl}\,a^{3/2}\,\sqrt{\omega_s}}\sqrt{\frac{\lambda-1}{3\,\lambda-1}}\,{\rm e}^{-iy} \simeq \frac{1}{2\times 3^{1/4}\,k^{3/2}}\frac{M_s}{M_{\rm Pl}}{\rm e}^{-iy}\,, \tag{59}$$

resulting in a scale invariant scalar spectrum

$$P_s = \frac{k^3}{2\,\pi^2}\,|\zeta|^2 = \frac{1}{4\,\sqrt{3}\,\pi^2}\left(\frac{M_s}{M_{\rm Pl}}\right)^2\,. \tag{60}$$

The tensor-to-scalar ratio for the primordial perturbations thus depends on the ratio of the two energy scales M_t and M_s through,

$$\frac{P_t}{P_s} = 8\,\sqrt{3}\,\left(\frac{M_t}{M_s}\right)^2\,. \tag{61}$$

C. Cubic Terms

In this subsection we consider nonlinear perturbations around the FRW background driven by the "dark matter as integration constant". We adopt the following metric ansatz.

$$N = 1, \quad N_i = \partial_i B + n_i, \quad g_{ij} = a^2 e^{2\zeta}\left(e^h\right)_{ij}\,, \tag{62}$$

where n_i is transverse and h_{ij} is transverse traceless: $\partial^i n_i = 0$, $\partial^i h_{ij} = 0$ and $h^i{}_i = 0$. Throughout this subsection, indices are raised and lowered by δ^{ij} and δ_{ij}. We consider ζ, B, n_i and h_{ij} as $O(\epsilon)$ and perform perturbative expansion with respect to ϵ.

In order to calculate the action up to cubic order, it suffices to solve the momentum constraint up to the first order. The momentum constraint at the first order is

$$\partial_i\left[(3\lambda-1)\,a^2\,\dot{\zeta} - (\lambda-1)\partial^2 B\right] + \frac{1}{2}\partial^2 n_i = 0, \tag{63}$$

leading to

$$B = \frac{3\lambda-1}{\lambda-1}\frac{\dot{\zeta}}{a^{-2}\partial^2}, \quad n_i = 0, \tag{64}$$

where $\partial^2 = \partial^i \partial_i$.

It is somewhat cumbersome but straightforward to calculate the kinetic action up to the third order. The result is

$$\begin{aligned} I_{kin} &= \frac{M_{Pl}^2}{2} \int N dt \sqrt{g} d^3\vec{x} (K^{ij}K_{ij} - \lambda K^2) \\ &= M_{Pl}^2 \int dt d^3\vec{x} a^3 \left[-\frac{3}{2}(3\lambda - 1)H^2 + \frac{3}{2}(3\lambda - 1)(2\dot{H} + 3H^2)\zeta \left(1 + \frac{3}{2}\zeta + \frac{3}{2}\zeta^2\right) \right. \\ &\qquad + (1+3\zeta)\left(a^{-2}\dot{\zeta}\partial^2 B + \frac{1}{8}\dot{h}^{ij}\dot{h}_{ij}\right) + \frac{1}{2}a^{-4}\zeta\partial^i(\partial_i B \partial^2 B + 3\partial^j B \partial_i \partial_j B) \\ &\qquad \left. + \frac{1}{2}(a^{-2}\partial^k h^{ij}\partial_k B - 3\dot{h}^{ij}\zeta)a^{-2}\partial_i\partial_j B - \frac{1}{4}a^{-2}\dot{h}^{ij}\partial_k h_{ij}\partial^k B \right] + O(\epsilon^4). \end{aligned} \tag{65}$$

The first term does not depend on the perturbation and the second term, which is proportional to $2\dot{H} + 3H^2$, vanishes because of the background equation of motion (27). Thus what we are interested in are the quadratic part $I_{kin}^{(2)}$ and the cubic part $I_{kin}^{(3)}$, where

$$\begin{aligned} I_{kin}^{(2)} &= M_{Pl}^2 \int dt d^3\vec{x} a^3 \left(a^{-2}\dot{\zeta}\partial^2 B + \frac{1}{8}\dot{h}^{ij}\dot{h}_{ij}\right), \\ I_{kin}^{(3)} &= M_{Pl}^2 \int dt d^3\vec{x} a^3 \left[3\zeta\left(a^{-2}\dot{\zeta}\partial^2 B + \frac{1}{8}\dot{h}^{ij}\dot{h}_{ij}\right) + \frac{1}{2}a^{-4}\zeta\partial^i(\partial_i B\partial^2 B + 3\partial^j B\partial_i\partial_j B) \right. \\ &\qquad \left. + \frac{1}{2}(a^{-2}\partial^k h^{ij}\partial_k B - 3\dot{h}^{ij}\zeta)a^{-2}\partial_i\partial_j B - \frac{1}{4}a^{-2}\dot{h}^{ij}\partial_k h_{ij}\partial^k B \right]. \end{aligned} \tag{66}$$

When B is eliminated by using (64), one can easily see that each term in $I_{kin}^{(3)}$ is marginal, i.e. has vanishing scaling dimension under the scaling (1), and that each coefficient remains of $O(1)$ (multiplied by the overall factor M_{Pl}^2) in the limit $\lambda \to \infty$.

As we have already calculated power spectra in the previous subsections, we know that the amplitudes of quantum fluctuations are

$$\langle h_{ij}\, h_{kl} \rangle \sim \left(\frac{M_t}{M_{Pl}}\right)^2, \quad \langle \zeta\, \zeta \rangle \sim \left(\frac{M_s}{M_{Pl}}\right)^2. \tag{67}$$

Thus, $I_{kin}^{(3)}$ is smaller than $I_{kin}^{(2)}$ and the perturbative expansion makes perfect sense if

$$M_t^2 \ll M_{Pl}^2, \quad M_s^2 \ll M_{Pl}^2 \tag{68}$$

in the UV with $\lambda \to \infty$. The same conclusion holds for all other terms in the action (7) since all terms which are not included in I_{kin} are independent of λ and are either marginal or relevant. The condition (68) is equivalent to (32) and should be considered as a nontrivial condition on properties of the RG flow in the vicinity of $\lambda = +\infty$ in the UV.

V. SUMMARY AND DISCUSSION

In this paper, we have studied the dynamics of the projectable Hořava-Lifshitz (HL) gravity with the $z = 3$ scaling in the ultraviolet (UV), focusing on the limit $\lambda \to \infty$. This limit for the parameter λ is a natural candidate for the UV fixed point of the renormalization group (RG) flow, if one forbids a ghost degree of freedom (appearing in the regime $1/3 < \lambda < 1$) and hopes that general relativity (GR) (having $\lambda = 1$) be recovered at low energy. Contrary to naive expectations, the system is well behaved in the limit $\lambda \to \infty$. Indeed, the dynamics can be even simpler due to the $1/\lambda$ suppression of the coupling between the gravity and matter sectors. We have analyzed tensor and scalar gravitons in the FRW universe driven by "dark matter as integration constant", and shown that the amplitudes of quantum fluctuations remain finite. The theory in the UV with $\lambda \to \infty$ is weakly coupled, provided that the condition (32) is satisfied.

While we have argued that the theory behaves well in the UV with the $\lambda \to \infty$ limit, cosmological implication of the result has not been explored yet. This is because of the lack of our understanding of the low energy dynamics with the $\lambda \to 1 + 0$ limit. This limit is the candidate for an infrared fixed point of the RG flow since GR has the value $\lambda = 1$.

It is known that in the limit $\lambda \to 1 + 0$, the scalar graviton gets strongly coupled. Strong coupling itself does not imply loss of predictability since all coefficients of infinite number of terms in the perturbative expansion can be written in terms of finite number of parameters in the action if the theory is renormalizable. However, the strong coupling implies breakdown of the perturbative expansion in the scalar graviton sector and, thus, we need nonperturbative analysis. For spherically-symmetric, static, vacuum configurations, it was shown by nonperturbative analysis that the limit $\lambda \to 1 + 0$ is indeed continuous and recovers GR [2]. This result may be considered as an analogue of Vainshtein effect and suggests the possibility that the scalar graviton may safely be decoupled from the rest of the world, i.e. the tensor graviton and the matter sector, in the limit $\lambda \to 1 + 0$. A similar work for super-horizon nonlinear cosmological perturbations in universes driven by "dark matter as integration constant" is in progress [12]. Nonetheless, it is fair to say that our understanding of the fate of the scalar graviton in the limit $\lambda \to 1 + 0$ is far from complete.

For this reason, we have not conducted a full analysis of cosmological implication (e.g. on the CMB spectrum) of the result of this paper.

Fortunately, the simple scenario in [10] does not suffer from the lack of our understanding of the $\lambda \to 1+0$ limit. For example, one can reliably calculate non-Gaussianities in cosmological perturbations [13]. A scalar field responsible for (almost) scale-invariant cosmological perturbations acts as a curvaton: it is sub-dominant at the time of sound horizon exit, later becomes dominant and finally reheats the universe. The only property of HL gravity needed for this mechanism is the anisotropic scaling with $z = 3$. (Thus, this mechanism should work also in other versions of HL gravity [5, 14].) When energy density of this scalar field is sub-dominant in the early epoch, it is expected that the only important effect of gravity to the dynamics of the scalar field is to provide an expanding background. Therefore, if λ runs towards 1 and GR is recovered during the epoch when the scalar field is sub-dominant, then the prediction of this scenario does not depend on details of the behavior of the scalar graviton in the limit $\lambda \to 1+0$.

An open issue regarding the scenario in [10] is to find a mechanism for Lorentz invariance restoration in the matter sector at low energies. Actually, this issue is shared by HL gravity itself: even if one omits Lorentz violating terms in the matter sector, these terms will be generated by radiative corrections from graviton loops. These terms may be under control provided that $M \ll M_{\rm Pl}$, where M is the scale at which the anisotropic scaling becomes important [15]. Another approach to this problem is to enforce a universal Lorentz breaking at all sectors at high energies, while supersymmetrizing the standard model ensures the restoration of the Lorentz symmetry at low energies [16, 17].

ACKNOWLEDGMENTS

Part of this work was done during YITP molecule-type workshop (T-10-05): Cosmological Perturbation and Cosmic Microwave Background. The authors thank YITP for stimulating atmosphere and warm hospitality. This work was supported by the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. S.M. acknowledges the support by Grant-in-Aid for Scientific Research 17740134, 19GS0219, 21111006, 21540278, by Japan-Russia Research Cooperative Program.

Appendix A: Example with scalar field

In Sec. III we have seen that coupling between gravity and matter sectors is suppressed by $1/\lambda$ and that these sectors decouple in the limit $\lambda \to \infty$. Motivated by this, in Sec. IV we have studied a pure gravity system and analyzed the evolution of perturbations around the FRW background driven by the "dark matter as integration constant". In this appendix, in order to justify this treatment, we consider a simple system of a scalar field in HL gravity and show that gravity and matter are indeed decoupled in the limit $\lambda \to \infty$ for linear perturbations.

We consider a single Lifshitz scalar field with the dynamical critical exponent $z = 3$, in accordance with the gravity sector. The dynamics of the field is described by the action

$$I_m = \frac{1}{2} \int dt\, d^3x\, N \sqrt{g} \left[\frac{1}{N^2} \left(\partial_t \varphi - N^i \partial_i \varphi \right)^2 + \varphi\, \mathcal{O}_\phi\, \varphi - 2\, V \right] , \tag{A1}$$

where the operator containing the gradients is defined as

$$\mathcal{O}_\phi \equiv \frac{1}{M_\phi^4} \left(D_i D^i \right)^3 - \frac{\kappa_\phi}{M_\phi^2} \left(D_i D^i \right)^2 + c_\phi^2\, D_i D^i \, . \tag{A2}$$

After decomposing the field into zero mode and perturbations as $\varphi = \phi + \delta\phi$, we vary the background action with respect to the scale factor and the field, to obtain the equations of motion

$$\begin{aligned} & -\frac{3\,\lambda - 1}{2} \left(2\, \dot{H} + 3\, H^2 \right) = \frac{1}{M_{\rm Pl}^2} \left(\frac{\dot{\phi}^2}{2} - V \right) , \\ & \ddot{\phi} + 3\, H\, \dot{\phi} + V' = 0 \, , \end{aligned} \tag{A3}$$

where the second (Klein-Gordon) equation is a special case of the energy-nonconservation equation (24), with $Q(t) = 0$. This extra information comes from specifying a field source for the perfect fluid description.

For perturbations, we calculate the quadratic action and expand modes in Fourier space, through

$$\delta(t, \vec{x}) = \frac{1}{(2\,\pi)^{3/2}} \int d^3k\, \mathrm{e}^{i\, \vec{k}\cdot\vec{x}}\, \hat{\delta}(t, \vec{k}) \, , \tag{A4}$$

where δ represents any scalar degree. The momentum constraint gives a relation for the nondynamical degree of freedom B,

$$\hat{B} = -\frac{a^2}{k^2} \left[\frac{3\,\lambda - 1}{\lambda - 1}\, \dot{\hat{\zeta}} + \frac{\dot{\phi}}{(\lambda - 1) M_{\rm Pl}^2}\, \delta\hat{\phi} \right] . \tag{A5}$$

After eliminating B by using this relation, the resulting action turns out to be a coupled system involving $\delta\phi$ and ζ. In order to analyze the system, we perform the following field redefinition

$$\psi \equiv a^{3/2} \begin{pmatrix} \delta\phi \\ \sqrt{\frac{2\,(3\,\lambda-1)}{\lambda-1}}\, M_{\rm Pl}\, \zeta \end{pmatrix} , \tag{A6}$$

to obtain the canonically normalized action

$$I = \frac{1}{2} \int dt\, d^3k \, \left(\dot{\hat{\psi}}^\dagger \dot{\hat{\psi}} + \dot{\hat{\psi}}^\dagger X \hat{\psi} - \hat{\psi}^\dagger X \dot{\hat{\psi}} - \hat{\psi}^\dagger \Omega^2 \hat{\psi} \right) , \tag{A7}$$

where $X = -X^T$ and $\Omega^2 = (\Omega^2)^T$ are both real matrices, with elements

$$X_{11} = X_{22} = 0, \quad X_{12} = -\frac{\dot{\phi}}{\sqrt{2\,(3\,\lambda-1)\,(\lambda-1)}\, M_{\rm Pl}} , \tag{A8}$$

$$\begin{aligned}
\left(\Omega^2\right)_{11} &= \frac{k^6}{M_\phi^4 a^6} + \frac{\kappa_\phi k^4}{M_\phi^2 a^4} + \frac{c_\phi^2 k^2}{a^2} - \frac{3\,V}{2\,M_{\rm Pl}^2\,(3\,\lambda-1)} - \frac{(9\lambda-1)}{4\,M_{\rm Pl}^2 (\lambda-1)(3\lambda-1)} \dot{\phi}^2 + V'' , \\
\left(\Omega^2\right)_{22} &= \frac{\lambda-1}{3\,\lambda-1} \left(\frac{k^6}{M_s^4 a^6} + \frac{\kappa_s k^4}{M_s^2 a^4} - \frac{k^2}{a^2} \right) + \frac{3}{2\,M_{\rm Pl}^2\,(3\,\lambda-1)} \left(\frac{\dot{\phi}^2}{2} - V \right) , \\
\left(\Omega^2\right)_{12} &= -\frac{V'}{\sqrt{2\,(3\,\lambda-1)\,(\lambda-1)}\, M_{\rm Pl}} .
\end{aligned} \tag{A9}$$

(For a general formalism to quantize coupled bosons, see e.g. [18].) For the action (A7), the couplings between the two degrees of freedom are suppressed by $1/\lambda$, and an initial adiabatic vacuum state can be defined unambiguously at early times

$$\psi_1 = \frac{M_\phi\, a^{3/2}}{\sqrt{2}\, k^{3/2}}\, {\rm e}^{-i \frac{k^3}{M_\phi^2} \int \frac{dt}{a^3}} , \qquad \psi_2 = \frac{3^{1/4}\, M_s}{\sqrt{2}\, k^{3/2}}\, {\rm e}^{-i \frac{k^3}{\sqrt{3}\, M_s^2} \int \frac{dt}{a^3}} . \tag{A10}$$

In other words, at leading order in $1/\lambda$ expansion, the gravity sector (ζ) is once again, decoupled from the matter sector ($\delta\phi$). In the UV regime with $\lambda \to \infty$, the solutions for both physical mode functions have constant amplitudes

$$\delta\phi = \frac{M_\phi}{\sqrt{2}\, k^{3/2}}\, {\rm e}^{-i \frac{k^3}{M_\phi^2} \int \frac{dt}{a^3}} , \qquad \zeta = \frac{M_s}{2 \times 3^{1/4}\, k^{3/2}\, M_{\rm Pl}}\, {\rm e}^{-i \frac{k^3}{\sqrt{3}\, M_s^2} \int \frac{dt}{a^3}} . \tag{A11}$$

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